INFLUENCE OF RIVETED STIFFENERS ON CRACK DEVELOPMENT AROUND A HOLE

V. N. Maksimenko

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When planning and analyzing sheet structure elements according to the "safe damage" principle, there is a need to estimate the influence of the reinforcing set on the stress intensity factor (SIF) at the crack apices. A survey of the research in this area, principally for an isotropic plate with one crack, can be found in [1, 2], say. In practice, there are no investigations denoted to crack development around a hole in plates with point-attached reinforcing elements.

On the basis of an asymptotic structureless theory of point connections, general representations are constructed below for solutions of this problem for an anisotropic plate with an elliptical hole. The problem is reduced to the combined solution of a system of singular equations in the unknown functions given on the crack contours and a system of linear algebraic equations in the forces being transmitted to the plate through the rivet. The influence is investigation of anisotropy of the plate material, the hole shape, the stiffness and location of the reinforcing set, the rib damage on the SIF at the crack apices. Appropriate results for isotropic plates are obtained by passing to the limit in the anisotropy parameters in the numerical solution.

1. Let us consider an anisotropic plate of constant thickness h weakened by an elliptical hole Λ with semiaxes a, b and a system of smooth nonintersecting curvilinear through slits (cracks) L_j ($j = \overline{1, k}$) (Fig. 1). The plate is strengthened by m rectilinear stiffener ribs (stringers) attached to the plate by using rivets.* If not especially stipulated, we consider all the slits internal.

Let us take the following simplifying assumptions recommended well in applications [1-3]: 1) a plane stress state is realized in the plate; 2) the reinforcing system of linear type stringers, and their attenuation because of rivet arrangements is not considered; 3) the plate and reinforcing elements interact with each other in one plane and only at the bracket points; 4) all the rivets are identical, the rivet radius (adhesion area) r is small as compared with their spacing and other characteristic dimensions; 5) when the crack passes through the rivet hole we do not take account of the influence of this hole and its being filled by the rivet; 6) we simulate the action of the rivet: in the stringer by the action of a concentrated force applied to a point corresponding to the center of the rivet in a continuous rib, and in the plate according to the structureless asymptotic theory of point connections [3] by the action of a concentrated force in the external zone and by the action of a concentrated force with a certain correction factor dependent on the kind of bracket in the near zone around the rivet. This zone has the order of a characteristic linear dimension of the adhesion area r; 7) each rivet is a linearly elastic spring connecting points of its axis of rotation belonging the fastening elements. The spring stiffness is identical in all directions and is known. The correction factor and the spring stiffness can be found either from the solution of the internal problem or experimentally [3, 4].

We limit ourselves to consideration of the case when an external load field in the plate, given by the forces $X_n + iY_n$ on the hole outline and σ_{ij}^0 at infinity, acts on an elastic system while concentrated forces $\overline{Q}_s^1 = Q_s^1 \exp[i(\vartheta_s + \pi)]$, $\overline{Q}_s^2 = Q_s^2 \exp(i\vartheta_s)$ are applied to the ends of the s-th rib. Here ϑ_s is the angle formed by the rib s with the axis x (see Fig. 1).

*We understand any technological operation or method of point fastening (welding, gluing, riveting, bolt connection, etc.) as a rivet connection, when the size of the adhesion area is small compared with the characteristic dimensions of the body and the bracket spacing.

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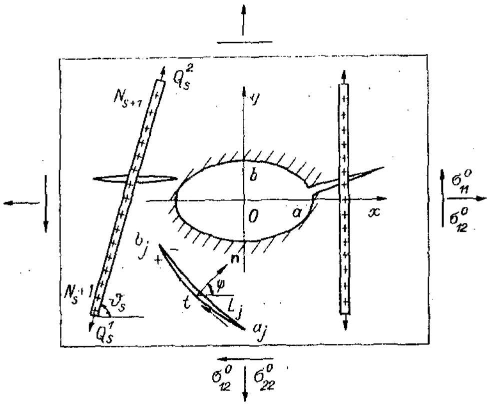

Fig. 1

Let N denote the total number of bracket points and $\tau^k = x^k + iy^k$, $\widetilde{P}_k = P_k \exp(i\alpha_k)$, respectively, their coordinates and the unknown forces being transmitted from the stiffeners to the plate therein. We select the numbering so that the rivets along the reinforcing element with number s are arranged in increasing numerical order starting with N_{s+1} to $N_s + n_s$ (n_s is the total number of rivets on the rib s), $N_{s+1} = N_s + n_s$, $N_1 = 0$, $N_{m+1} = N$. According to assumption 2, we have $\alpha_k = \vartheta_s$ $(N_s < k < N_{s+1},\ s = \overline{1, m})$. The stress and displacement in the plate can be expressed in terms of two analytic functions $\Phi_\nu(z_\nu)$ [5]:

$$
\begin{gathered}
(\sigma_x, \tau_{xy}, \sigma_y) = 2 \operatorname{Re}\left\{ \sum_{\nu=1}^{2} \left(\mu_\nu^2, -\mu_\nu, 1\right) \Phi_\nu(z_\nu) \right\}, \\
(u, v) = 2 \operatorname{Re}\left\{ \sum_{\nu=1}^{2} \left(p_\nu, q_\nu\right) \varphi_\nu(z_\nu) \right\}, \\
z_\nu = x + \mu_\nu y, \quad p_\nu = a_{11}\mu_\nu^2 - a_{16}\mu_\nu + a_{12}, \quad q_\nu = a_{12}\mu_\nu + a_{22}\mu_\nu^{-1} - a_{26},
\end{gathered}
\tag{1.1}
$$

where $\varphi_\nu(z_\nu)$ is the primitive for $\Phi_\nu(z_\nu)$, a_{ij} are the strain coefficient from Hooke's law, and μ_ν are roots of the characteristic equation.

Taking account of the simplifying assumptions and the superposition principle, we seed the function (1.1) in the form

$$
\Phi_\nu(z_\nu) = \sum_{j=1}^{3} \Phi_\nu^j(z_\nu). \tag{1.2}
$$

Here $\Phi_\nu^1(z_\nu)$ is the solution for an unreinforced plate with hole Λ without cracks which satisfies the boundary conditions on the hole outline and at infinity and can be determined by using known methods [5].

Using the solution of the problem on the action of a unit concentrated force $\exp(i\varphi)$ at a point τ of an infinite anisotropic plate with elliptical hole Λ free from external forces [6]

$$
\begin{gathered}
\Psi_\nu(z_\nu, \tau_\nu, \varphi) = \frac{1}{\omega_\nu'(\zeta_\nu)} \left\{ \frac{A_\nu(\varphi)}{\zeta_\nu - \eta_\nu} + \frac{l_\nu \overline{A_1(\varphi)}}{\zeta_\nu(\zeta_\nu \bar{\eta}_1 - 1)} + \frac{n_\nu \overline{A_2(\varphi)}}{\zeta_\nu(\zeta_\nu \bar{\eta}_2 - 1)} \right\}, \\
z_\nu = \omega_\nu(\zeta_\nu) = \frac{a - i\mu_\nu b}{2}\zeta_\nu + \frac{a + i\mu_\nu b}{2}\frac{1}{\zeta_\nu}, \quad |\zeta_\nu| > 1, \\
\zeta_\nu = \zeta_\nu(z_\nu) = \frac{z_\nu + \sqrt{z_\nu^2 - (a^2 + \mu_\nu^2 b^2)}}{a - i\mu_\nu b}, \quad \zeta_\nu(\infty) = \infty, \\
l_\nu = \frac{\bar{\mu}_{3-\nu} - \bar{\mu}_1}{\mu_\nu - \mu_{3-\nu}}, \quad n_\nu = \frac{\bar{\mu}_{3-\nu} - \bar{\mu}_2}{\mu_\nu - \mu_{3-\nu}}
\end{gathered}
\tag{1.3}
$$

and taking account of interaction of inclusions asymptotically by using the superposition principle, we set (see assumption 6)

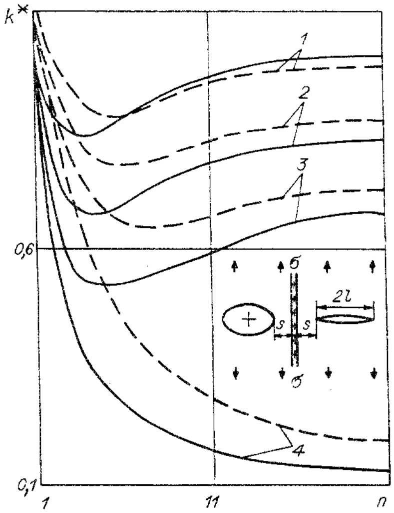

Fig. 2

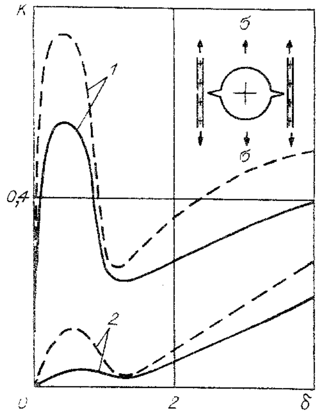

Fig. 3

$$\Phi_\nu^2(z_\nu) = \sum_{k=1}^{N} P_k \Psi_{\nu k}(z_\nu). \tag{1.4}$$

We will consider that $\Psi_{\nu k}(z_\nu) = \Psi_\nu(z_\nu, \tau_\nu^k, \alpha_k)$ everywhere except in the near zone at the k-th rivet [3].

An expression is later required for the displacement of the bracket points τ^k in the plate in terms of the potential $\varphi_\nu(z_\nu)$. In this case we will understand the expression $\psi_{\nu k}(\tau_\nu^k)$ to be the arithmetic mean of values of the function $\psi_{\nu k}(z_\nu)$ at points z_ν^p corresponding to the points $z^p = \tau^k + \Delta r \exp[i(\alpha_k + \pi p)]\ (p = 0, 1)$ under the affine mapping $z_\nu = \mathrm{Re}(z) + \mu_\nu \mathrm{Im}(z)$. Here $\psi_{\nu k}(z_\nu)$ is the primitive function of $\Psi_{\nu k}(z_\nu)$; and Δ is the correction factor (see assumption 6) that depends on the rivet material and its construction. The dependence of the force in the rivet on Δ turns out to be sufficiently weak [3], and, consequently, the errors in determining Δ has no substantial influence on the final result.

We seek the function $\Phi_\nu^3(z_\nu)$ in the form of generalized Cauchy integrals whose kernels are fundamental solutions (1.3):

$$\begin{gathered}\Phi_\nu^3(z_\nu) = \frac{1}{\omega_\nu'(\zeta_\nu)} \int_L \left\{ \frac{\Omega_\nu^*(\tau)}{\zeta_\nu - \eta_\nu} + \frac{l_\nu \overline{\Omega_1^*(\tau)}}{\zeta_\nu(\zeta_\nu \overline{\eta}_1 - 1)} + \frac{n_\nu \overline{\Omega_2^*(\tau)}}{\zeta_\nu(\zeta_\nu \overline{\eta}_2 - 1)} \right\} ds = \\ = \frac{1}{2\pi i \omega_\nu'(\zeta_\nu)} \int_L \left\{ \frac{\Omega_\nu(\tau)\, d\tau_\nu}{\eta_\nu - \zeta_\nu} + \frac{l_\nu \overline{\Omega_1(\tau)\, d\tau_1}}{\zeta_\nu(\zeta_\nu \overline{\eta}_1 - 1)} + \frac{n_\nu \overline{\Omega_2(\tau)\, d\tau_2}}{\zeta_\nu(\zeta_\nu \overline{\eta}_2 - 1)} \right\}, \\ \Omega_\nu(t) = -2\pi i \Omega_\nu^*(t)/M_\nu(t), \quad M_\nu(t) = \mu_\nu \cos\psi - \sin\psi,\end{gathered} \tag{1.5}$$

where $\Omega_\nu(t) = \{\Omega_{\nu j}(t) | t \in L_j;\ j = \overline{1, k}\}$ are unknown complex functions on L, and $d\tau_\nu = M_\nu(\tau) ds$ (ds is the arc length element of L), $\psi = \psi(t)$ is the angle between the normal **n** to L and the point t on the x axis. We direct the normal to the right in the positive traversal of L (see Fig. 1).

The functions $\Phi_\nu^j(z_\nu)\ (j = 2, 3)$ thus constructed satisfy the conditions $X_n = Y_n = 0$ on Λ and damp out at infinity. Therefore, selection of $\Phi_\nu(z_\nu)$ in the form (1.2) automatically assumes satisfaction of the external loading conditions everywhere except on L.

2. The boundary conditions at the slits L free from external fores can be given the form [7]

$$\begin{gathered} a(t)\Phi_1^\pm(t_1) + b(t)\overline{\Phi_1^\pm(t_1)} + \Phi_2^\pm(t_2) = 0, \\ a(t) = a_0 \frac{M_1(t)}{M_2(t)},\ b(t) = b_0 \frac{\overline{M_1(t)}}{M_2(t)},\ a_0 = \frac{\mu_1 - \overline{\mu}_2}{\mu_2 - \overline{\mu}_2},\ b_0 = \frac{\overline{\mu}_1 - \overline{\mu}_2}{\mu_2 - \overline{\mu}_2} \end{gathered} \tag{2.1}$$

(the plus (minus) refers to the left (right) edge of L).

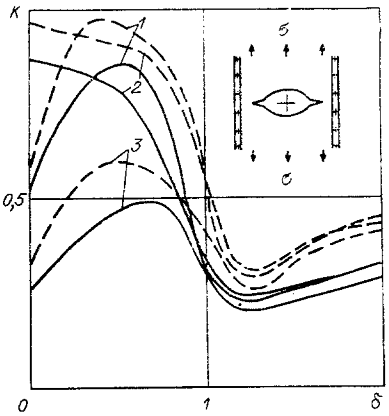

Fig. 4

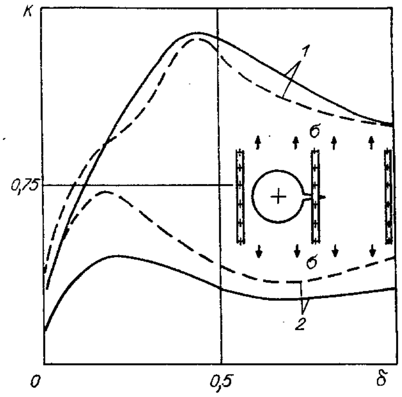

Fig. 5

Substituting the limit values of the functions $\Phi_\nu(z_\nu)$ from (1.2) and (1.4) into (2.1), we obtain ($\Omega(t) = \Omega_1(t)$)

$$\int_L \frac{\Omega(\tau)\,d\tau_1}{\eta_1-\zeta_1} + \int_L \{K_1(t,\tau)\Omega(\tau) + K_2(t,\tau)\overline{\Omega(\tau)}\}\,ds + \sum_{k=1}^{m} c_k(t)P_k = f(t),$$

$$K_1(t,\tau)\,ds = \frac{1}{2\overline{b(t)}}\left\{\frac{\overline{b(\tau)}-\overline{b(t)}}{\overline{\eta}_2-\overline{\zeta}_2}\,d\overline{\eta}_2 + \overline{b(t)}\,d\ln\frac{\overline{\eta}_2-\overline{\zeta}_2}{\eta_1-\zeta_1} - \frac{\overline{l}_1\overline{a(t)}\,d\tau_1}{\omega_1'(\zeta_1)\overline{\zeta}_1(\overline{\zeta}_1\eta_1-1)} + \right.$$

$$+\frac{\overline{n}_1 a(\tau)\overline{a(t)}\,d\tau_2}{\omega_1'(\zeta_1)\overline{\zeta}_1(\overline{\zeta}_1\eta_2-1)} - \frac{n_1\overline{b(t)}\,\overline{b(\tau)}\,d\overline{\tau}_2}{\omega_1'(\zeta_1)\zeta_1(\zeta_1\overline{\eta}_2-1)} - \frac{\overline{l}_2\,d\tau_1}{\omega_2'(\zeta_2)\overline{\zeta}_2(\overline{\zeta}_2\eta_1-1)} +$$

$$\left. + \frac{\overline{n}_2 a(\tau)\,d\tau_2}{\omega_2'(\zeta_2)\overline{\zeta}_2(\overline{\zeta}_2\eta_2-1)}\right\}, \tag{2.2}$$

$$K_2(t,\tau)\,ds = \frac{1}{2\overline{b(t)}}\left\{\overline{a(t)}\,d\ln\frac{\overline{\eta}_2-\overline{\zeta}_2}{\eta_1-\overline{\zeta}_1} + \frac{\overline{a(\tau)}-\overline{a(t)}}{\overline{\eta}_2-\overline{\zeta}_2}\,d\overline{\eta}_2 + \frac{\overline{n}_1\overline{a(t)}\,b(\tau)\,d\tau_2}{\omega_1'(\zeta_1)\overline{\zeta}_1(\overline{\zeta}_1\eta_2-1)} + \right.$$

$$\left. + \frac{l_1\overline{b(t)}\,d\overline{\tau}_1}{\omega_1'(\zeta_1)\zeta_1(\zeta_1\overline{\eta}_1-1)} - \frac{n_1\overline{b(t)}\,\overline{a(\tau)}\,d\overline{\tau}_2}{\omega_1'(\zeta_1)\zeta_1(\zeta_1\overline{\eta}_2-1)} + \frac{\overline{n}_2 b(\tau)\,d\tau_2}{\omega_2'(\zeta_2)\overline{\zeta}_2(\overline{\zeta}_2\eta_2-1)}\right\},$$

$$c_k(t) = \frac{\pi i}{\overline{b(t)}}\{\overline{a(t)}\,\overline{\Psi_{1k}(t_1)} + \overline{b(t)}\,\Psi_{1k}(t_1) + \overline{\Psi_{2k}(t_2)}\},$$

$$f(t) = \frac{\pi i}{\overline{b(t)}}\{\overline{a(t)}\,\overline{\Phi_1^1(t_1)} + \overline{b(t)}\,\Phi_1^1(t_1) + \overline{\Phi_2^1(t_2)}\}.$$

The kernels $K_p(t, \tau)$ ($p = 1, 2$) are continuous according to the assumptions relative to L.

It is necessary to supplement (2.2) with conditions for uniqueness of the displacements during traversal of each crack L_j:

$$\int_{L_j} \Omega(\tau)\,d\tau_1 = 0 \qquad (j = \overline{1, k}). \tag{2.3}$$

Let $u(z, \vartheta)$ denote the plate displacement at the point z in the direction of the vector $\exp(i\vartheta)$. Then from the compatibility conditions for the rib and plate displacements at the bracket points $u(\tau^{j+1}, \vartheta_s) - u(\tau^j, \vartheta_s) = \frac{|\tau^{j+1}-\tau^j|}{E^sF^s}\left\{Q_s^2 - \sum_{k=j+1}^{N_{s+1}} P_k\right\} - q(P_{j+1}-P_j),\quad j = \overline{N_s+1, N_{s+1}-1}\ \ (s = \overline{1, m})$

and the stiffener equilibrium conditions we find the missing N equations to determine the forces P_k:

$$\sum_{k=1}^{N} c_{jk}P_k + \mathrm{Re}\left\{\int_L K_{1j}(\tau)\Omega(\tau)\,ds\right\} = f_j, \tag{2.4}$$

$$K_{1j}(\tau)\,ds = \frac{1}{\pi i}\left\{\Delta_1(\vartheta_s)\ln\frac{\eta_1-\zeta_1^{j+1}}{\eta_1-\zeta_1^j}d\tau_1 - a(\tau)\Delta_2(\vartheta_s)\ln\frac{\eta_2-\zeta_2^{j+1}}{\eta_2-\zeta_2^j}d\tau_2 + \right.$$
$$+\overline{b(\tau)}\sum_{\nu=1}^{2}\Delta_\nu(\vartheta_s)n_\nu\ln\frac{(\zeta_\nu^{j+1}\overline{\eta}_2-1)\zeta_\nu^j}{(\zeta_\nu^j\overline{\eta}_2-1)\zeta_\nu^{j+1}}d\overline{\tau}_2+\overline{b(\tau)}\,\overline{\Delta_2(\vartheta_s)}\,\overline{\ln\frac{\eta_2-\zeta_2^{j+1}}{\eta_2-\zeta_2^j}}d\overline{\tau}_2+$$
$$\left.+\sum_{\nu=1}^{2}\overline{\Delta_\nu(\vartheta_s)}\,\overline{l}_\nu\,\overline{\ln\frac{(\zeta_\nu^{j+1}\overline{\eta}_1-1)\zeta_\nu^j}{(\zeta_\nu^j\overline{\eta}_1-1)\zeta_\nu^{j+1}}}d\tau_1 - a(\tau)\sum_{\nu=1}^{2}\overline{\Delta_\nu(\vartheta_s)}\,\overline{n}_\nu\,\overline{\ln\frac{(\zeta_\nu^{j+1}\overline{\eta}_2-1)\zeta_\nu^j}{(\zeta_\nu^j\overline{\eta}_2-1)\zeta_\nu^{j+1}}}d\tau_2\right\},$$
$$c_{jk} = 2\,\mathrm{Re}\left\{\sum_{\nu=1}^{2}\Delta_\nu(\vartheta_s)\left[\psi_{\nu k}(\tau_\nu^{j+1})-\psi_{\nu k}(\tau_\nu^j)\right]\right\}+q(\delta_{j+1,k}-\delta_{jk})+$$
$$+\frac{|\tau^{j+1}-\tau^j|}{E^sF^s}H(k-j),\quad H(x)=\begin{cases}0, & x\leqslant 0,\\ 1, & x>0,\end{cases}$$
$$f_j=\frac{|\tau^{j+1}-\tau^j|}{E^sF^s}Q_s^2-2\mathrm{Re}\left\{\sum_{\nu=1}^{2}\Delta_\nu(\vartheta_s)\left[\varphi_\nu^1(\tau_\nu^{j+1})-\varphi_\nu^1(\tau_\nu^j)\right]\right\},$$
$$j=\overline{N_s+1,\,N_{s+1}-1}\qquad(s=\overline{1,m});$$
$$\sum_{j=N_s+1}^{N_{s+1}}P_j=Q_s^2-Q_s^1\qquad(s=\overline{1,m}).\tag{2.5}$$

Here E^s, F^s are the Young's modulus and cross-sectional area of the s-th rib, δ_{jk} is the Kronecker delta, and q is the pliability of the springs simulating the rivets. Sample formulas to determine q are given in [4], say, on the basis of experimental data.

Limit cases of the problem under consideration are: a) reinforced half-plane ($x > 0$) with a system of curvilinear slits ($b/a \to \infty$); b) reinforced infinite plate with a rectilinear slit $\Lambda = \{|x| < a;\ y = 0\}$ and a system of curvilinear slits L_j ($b/a = 0$).

3. According to the assumptions relative to the slits L_j ($j = \overline{1, k}$), (2.2) is a singular integral equation in $\Omega(t)$. Its index is +1. The solution of (2.2) under the additional constraints (2.3) in the class of functions

$$\Omega(t)=\chi^j(\beta)(1-\beta^2)^{-1/2},\ t=\tau^j(\beta)\in L_j,\tag{3.1}$$

where $\chi^j(\beta)$ are bounded, Hölder-continuous functions in [−1, 1], exists and is unique [8].

By using the Gauss–Chebyshev quadrature formulas we reduce the solution of (2.2)-(2.5) to the solution of a system of linear algebraic equations in P_k ($k = \overline{1, N}$) and approximate values of the desired functions $\chi^j(\beta)$ $(j=\overline{1,k})$ at the Chebyshev nodes $\beta_i = \cos((2i-1)\pi/(2M))$ $(i=\overline{1,M})$ (see [9], say).

If the system (2.2)-(2.5) is solved and the values $\chi^j(\pm 1)$ are determined, then by using the asymptotic formulas $\Phi_\nu(z_\nu)\approx \pm 2^{-3/2}\chi_\nu^j\{\pm\dot{\tau}_\nu^j(\mp 1)/(z_\nu-c_\nu)\}^{1/2}(\nu=1,\,2)$, $\chi_1^j=\chi^j(\mp 1)$, $\chi_2^j=-a(c)\ \chi_1^j - b(c)\ \overline{\chi}_1^j$, $\dot{\tau}_\nu^j=\frac{d\tau_\nu^j}{d\beta}$ we find the stress distribution and the SIF of the separation and shear stresses $k_1(c)=\lim\limits_{t\to c}\sigma_n\sqrt{2\pi r}$, $k_2(c)=\lim\limits_{t\to c}\tau_n\sqrt{2\pi r}$ at the apices $c=\tau^j(\mp 1)$ of the cracks L_j. Here t is a point on the continuation of the crack beyond the tip c along the tangent; $r = |t - c|$.

If the slit L_p emerges, say, with the tip $a^p = \tau^p(-1)$ at the controls of the hole Λ, then the condition of uniqueness of the displacement (2.3) for j = p should be discarded and the corresponding potential (1.5) refined. According to [9], we execute the refinement by using the condition of boundedness of the solution at the point a^p. Consequently

$$\int_{L_p}\{[P_\nu(z,\tau)-P_\nu(z,a^p)]\,\Omega_{1p}(\tau)\,d\tau_1+[Q_\nu(z,\tau)-Q_\nu(z,a^p)]\,\overline{\Omega_{1p}(\tau)}\,d\overline{\tau}_1\},$$
$$P_1(z,\tau)=\frac{1}{\eta_1-\zeta_1}-\frac{1}{\zeta_1(1-\zeta_1\overline{\eta}_2)},\quad Q_1(z,\tau)=\frac{a_0}{\zeta_2(1-\zeta_2\overline{\eta}_2)}-\frac{a_0}{\eta_2-\zeta_2},$$
$$P_2(z,\tau)=\frac{l_1}{\zeta_1}\left(\frac{l_1}{1-\zeta_1\overline{\eta}_1}-\frac{1}{1-\zeta_1\overline{\eta}_2}\right),\ Q_2(z,\tau)=\frac{1}{\zeta_2}\left(\frac{1}{1-\zeta_2\overline{\eta}_1}-\frac{n_2\overline{a}_0}{1-\zeta_2\overline{\eta}_2}\right)-\frac{l_0}{\eta_2-\zeta_2}.$$

This will result in corresponding changes in the integrals in L_p in (2.2) and (2.4).

TABLE 1

δ	M=8	M=16	[10]	δ	M=8	M=16	[10]
0,01	3,288	3,292	3,291	0,4	1,883	1,885	1,884
0,04	3,092	3,096	3,095	1	1,305	1,306	1,306
0,06	2,976	2,979	2,978	4	1,030	1,031	1,030
0,1	2,769	2,772	2,771	10	0,779	0,780	—

The numerical solution of the equations that occur on the edge slit L_p will be sought, as before, in the class of functions (3.1). To close the obtained system of algebraic equations we will take the additional condition $\chi^p(-1) = 0$ that eliminates the singularity of the function $\Omega(t)$ at the point a^p. Such a simplified method yields completely satisfactory results for internal apices of edge cracks [9].

4. Presented below for uniform tension by forces $\sigma_y^\infty = \sigma$ are certain results of computing the separation SIF at crack apices around an elliptical hole Λ in glass-epoxy composite plate (E_1 = 53.84 GPa, E_2 = 17.95 GPa, G_{12} = 8.63 GPa, ν_1 = 0.25) and isotropic material (ν = 0.33) reinforced along the line $x = x_k$ identical stringers free from external forces. The bracket points are arranged symmetrically relative to the x axis with constant spacing p. The x axis passes midway between the rivets and the number of rivets on each stringer is identical and equal to 2n. The pliability of the bracket q was taken equal to zero, r/a = 0.025, p/a = 0.5 and the correction factor is Δ = 1. Data for an isotropic material are obtained by passing to the limit in the anisotropy parameters in the numerical solution.

Let an anisotropic plate (the principal direction of anisotropy E_1 of the plate material makes the angle $\varphi = 0$ with the x axis) contain an internal crack $L = \{\tau(\beta) = a + 2s + l(1 + \beta) \| \beta | < 1\}$ and be reinforced by one stringer passing through the middle of a connector (x_1 = a + s). Presented in Fig. 2 are magnitudes of the separation correction SIF at the left apex of the crack $k^* = k_1(-1)/(\sigma\sqrt{\pi l})$ as a function of n for m = (a − b)/(a + b) = 0 (circular hole), ℓ/a = 1 and different values of the relative rib stiffness $U = E_1ah/E^1F^1$ = 1, 0.5, 0.25, 0 (curves 1-4, respectively). The solid (dashed) lines refer to s/a = 0.1(0.5). For n > 15 the values of k^* vary insignificantly, i.e., no development can be achieved as the number of bracket points increases. The effectiveness of the reinforcement diminishes noticeably as U and the bracket spacing p increase.

Let one or two cracks $L_{1,2} = \{\tau^{1,2}(\beta) = \pm[a + l(1 + \beta)] \| \beta | < 1\}$ from the outline of a hole Λ in an anisotropic reinforced plate.

Represented in Figs. 3 and 4 is the dependence of the separation correction SIF for two cracks $L_{1,2}$ and two reinforcing elements ($x_{1,2} = \pm 2a$) at the apices of the cracks $K = k_1(1)/k_1^*$ ($k_1^* = \sigma\sqrt{\pi(a + 2l)}$ is the separation SIF at the vertices of an equivalent crack $L^* = \{|x| < a + 2l;\ y = 0\}$ in an unreinforced plate) on $\delta = 2\ell/a$ for m = 0, U = 0.25, 0 (curves 1 and 2) and U = 0.25, m = 0, 1, −0.9 (curves 1-3), respectively. Here and henceforth, the solid (dashed) lines refer to $\varphi = 0$ ($\pi/2$), n = 9. For small cracks ($\delta < 1$)K depends substantially on the degree of material anisotropy E_1/E_2 and the hole shape. For $\delta > 2$ the influence of the hole shape on K is weak. As E_1/E_2 increases, the difference between the values of K for $\varphi = 0$ and $\pi/2$ grows. Even for strongly prolate ellipses with a small opening the value of from the separation SIF for an equivalent crack in a plate with the same reinforcement. Therefore, the defects being considered cannot always be replaced by equivalent cracks, as is done in practice, in computations of reinforced plates. The minimal value of K is reached for cracks setting at approximately 0.25a behind the reinforcing element.

Presented in Fig. 5 are values of $K = k_1(1)/k_1^*$ $(k_1^* = \sigma\sqrt{\pi(a + l)})$ for one edge crack and three reinforcing elements ($x_{1,2} = \pm 1.4a$, $x_3 = 4.2a$, U = 0.5), 1) a stringer with number 1 is fractured along the middle, 2) undamaged stringer. The value of $k_1(1)$ in the case of a damaged reinforcement is increased considerably and becomes even higher than the corresponding values of k_1 for an unreinforced plate until the crack turns out to be in the zone of influence of the adjacent stringer. Therefore, reinforcement damage makes a structure more susceptible to fracture.

Computations indicate the efficiency of the selected representations and the computation algorithm. The values of k_1 in Figs. 2-5 are in agreement in the first three significant figures for $M \geq 16$. For instance, given for conversion in Table 1 are values of $k_1/(\sigma\sqrt{\pi\ell}$ at the apex of an edge crack L_1 starting from the contour of a circular hole in an unreinforced isotropic plate (in the computations it was assumed that $\mu_1 = 0.98i$ and $\mu_2 = 1.02i$) for M = 8.16 and appropriate values from [10].

For m = 1 (a hole is generated in the slit) the results of computations for an anisotropic material agree with the data of [11] while for an isotropic material they are in good agreement with results of a computation and experiment [4, 12]. The maximal error in determining k_1 with respect to the computational results [4] where the structure theory of rivets was used, is ~2%.

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